Hollow Cathodes as Electron Emitting Plasma Contactors: Theory and Computer Modeling

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Several researchers have suggested using hollow cathodes as plasma contactors for electrodynamic tethers, particularly to prevent the shuttle orbiter from charging to large negative potentials. Previous studies have shown that fluid models with anomalous scattering can describe the electron transport in hollow cathode-generated plasmas. An improved theory of the hollow cathode plasmas is developed, and computational results using the theory are compared with laboratory experiments. Numerical predictions for a hollow cathode plasma source of the type considered for use on the shuttle are presented, as are three-dimensional NASCAP/LEO calculations of the emitted ion densities and the resulting potentials in the vicinity of the orbiter. The computer calculations show that the hollow cathode plasma source makes vastly superior contact with the ionospheric plasma compared with either an electron gun or passive ion collection by the orbiter.

Nomenclature

e = electron charge
E = electric field
$f_1(v)$ = perturbed electron velocity distribution
j = electron current density
j_{th} = one-sided electron thermal current density
m = electron mass
n = plasma density
v = velocity
v_g = wave group velocity
V = keeper electrode voltage
W = wave energy density
α = measure of turbulence
ϵ_o = permittivity of free space
η = electrical resistivity
θ = electron temperature
κ = thermal conductivity
ν = scattering frequency
σ = electrical conductivity
ϕ = plasma potential
ϕ_b = barometric contribution to plasma potential
ϕ_j = current driven contribution to plasma potential
ω_p = plasma frequency

Introduction

HOLLOW cathode plasma sources have long been recognized as copious electron emitters.[1-4] Several researchers[5-10] have suggested using hollow cathodes as plasma contactors for electrodynamic tethers, particularly to prevent the shuttle orbiter from being driven to large negative potentials. Previous studies[11] have shown that fluid models with anomalous scattering can describe the electron transport in hollow cathode-generated plasmas. Improvements in the understanding of electron energy transfer near the hollow cathode orifice have led to improved theoretical models that compare well with laboratory hollow cathode results. These models have been incorporated in NASCAP/LEO,[12] a computer program that simulates in three dimensions the interaction of large, high-power spacecraft with the plasma environment found in low earth orbit. The calculations show that, by using a hollow cathode, the potential difference between the orbiter and the surrounding plasma can be maintained at tens of volts for tether currents of up to one ampere. This is in contrast to electron guns, which require accelerating potentials of thousands of volts, or to ion collection, which NASCAP/LEO calculations show requires potentials in excess of that generated by the tether. The high potentials needed in electron guns lead to excessive energy losses and leakage current problems, along with difficulties in maintaining the shuttle ground near plasma ground. From experience with geosynchronous spacecraft, it is known that the hollow cathode-generated plasma reduces differential charging on a spacecraft,[13] which would further reduce the risk of arcing during tether operation.

Theory of a Hollow Cathode Plasma

A schematic of the relevant parts of a hollow cathode is shown in Fig. 1. For the purposes of this paper, a hollow cathode is a cavity containing electrons, ions, and neutrals. At one end is an orifice through which the electrons, ions, and neutrals may exit. Outside the cathode, a keeper ring that is biased positively with respect to the cathode body accelerates the electrons. Near the keeper ring, energetic electrons ionize the neutrals. Most of the electrons stripped from the neutrals or emitted from the orifice escape to infinity. These are the electrons that are "emitted" by the cathode. Although most of the electrons escape to infinity, some are absorbed by the keeper ring. The ions that are emitted by the cathode or move from the vicinity of the keeper ring toward the cathode are quickly absorbed by the cathode body, so there is no significant plasma density between the cathode body and the keeper ring. The ions formed on the far side of the keeper are ac-

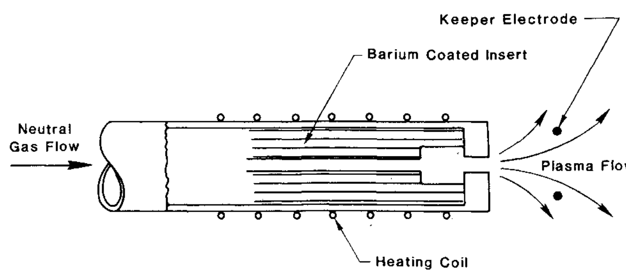

Fig. 1 Schematic diagram of a hollow cathode configuration.

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celerated to infinity. One alternative to this basic design is to confine the electrons to the immediate region of the cathode by means of a magnetic field. The electrons eventually leave the region, but because they are temporarily confined to this region, the probability of an ionizing collision is much greater. This results in much higher ion densities near the orifice and in a more efficient use of the neutral gas. The cathode referenced later in this paper is of this design. It ionizes about 10% of the gas.

The plasma in the vicinity of the cathode is described in terms of a thermal fluid of electrons and streaming ions moving under the electric field forces. The plasma is assumed to be quasineutral. Since there are no sources or sinks of charge other than the cathode itself, the total current j obeys the conservation of charge equation:

$$\nabla \cdot j = 0 \qquad \text{(Charge) (1a)}$$

The electron fluid is described with the standard momentum and energy conservation equations including convection:[14]

$$\nabla(n\theta) - n\,\nabla\phi = \eta n j \qquad \text{(Momentum) (1b)}$$

$$\nabla\cdot(\kappa\nabla\theta) - (5/2)\,j\cdot\nabla\theta + \eta j^2 = 0 \qquad \text{(Energy) (1c)}$$

To obtain these equations we assume that the electron drift velocity is small compared to the random thermal velocity and that the kinetic energy flux can be neglected.

The electric conductivity σ, the electrical resistivity η, and the thermal conductivity κ are calculated in terms of an effective scattering frequency, which is obtained from considerations of anomalous transport. The transport coefficients are

$$\eta^{-1} = \sigma = \epsilon_o \omega_p^2/\nu \qquad \text{(Electrical) (2a)}$$

and

$$\kappa = (3/2)\,\sigma\theta \qquad \text{(Thermal) (2b)}$$

In the absence of turbulence, the effective scattering frequency is just the electron-electron collision rate. In a turbulent plasma, the effective collision frequency is the rate at which electrons scatter from the random fluctuations of the electrostatic fields. The symbol ν can be written as

$$\nu = \omega_p \alpha \qquad (3)$$

The scattering frequency calculation is based on published[14] results of Langmuir wave generation by electron beam injection at a plasma boundary. The electrons emitted from the orifice and accelerated by the keeper form an electron beam. These electrons contribute to both the keeper current and the current that passes to infinity. The electron beam is unstable in the plasma, and this instability pumps electrostatic oscillations. We assume that at the injection boundary (the keeper ring) the waves are fully saturated. Part of the injected-beam kinetic energy flux is deposited in the plasma in the form of electrostatic waves; the rest remains beam energy. For simplicity we only evaluate this energy balance in one dimension, since the width of the deposition region is small (on the order of 10 to 100 Debye lengths). If we assume that the energy involved in the drift of the bulk plasma is small compared to the total energy, the energy flux balance equation is

$$jV = v_g W + \int \mathrm{d}v\, f_1(v)\, \frac{1}{2}\, m v^3 \qquad (4)$$

The current j is related to f_1 by

$$j = e \int \mathrm{d}v\, f_1(v) v \qquad (5)$$

Since some of the current is absorbed by the keeper, the appropriate j is greater than the electron current to infinity but less than the sum of the emitted and keeper currents. With the assumption that f_1 is flat between $v = 0$ and the velocity corresponding to the injection voltage, we get

$$jV = (1/2)\,jV + v_g W \qquad (6)$$

One-half jV is the energy flux associated with the drift velocity of the electrons into the plasma. The basic equations, Eqs. (1a-1c), do not explicitly include drift kinetic energy; it is assumed to be converted immediately to heat, which is included. The boundary condition at the minimum radius is that the heat flux into the first zone is one-half the kinetic energy of the injected beam.

Equation (6) gives

$$W = jV/2v_g \qquad (7)$$

The wave energy W is evenly divided between the energy in the periodic motion of the particles and the energy in the fluctuation of the electric field.[15] In order to evaluate the field energy, the group velocity of the waves is estimated to be equal to the thermal velocity of the electrons. The effective electron scattering frequency is taken to be[15]

$$\nu = \omega_p\,(\epsilon_o \langle E^2\rangle/2n\theta) \qquad (8)$$

These waves build up to the saturation level within several Debye lengths of the keeper ring; at greater distances the wave energy decreases by divergence and decay. We use the expression in Eq. (8) to evaluate the peak scattering frequency. Away from the source we assume that ν/ω_p decays. These considerations give the following expression for the peak value of the effective scattering frequency.

$$\nu = \omega_p \frac{1}{4}\frac{j}{j_{\text{th}}}\frac{V}{\theta} \qquad (9)$$

One-Dimensional Model of the Local Plasma

Near the hollow cathode, the ions and electrons expand primarily spherically outward. The plasma parameters vary primarily in the radial direction, and the problem can be treated as one-dimensional. This assumption leads to a simplification of the equations. We take the decay of the ratio of the wave energy to the particle energy to be an inverse square law. An inverse square dependence with a peak region of finite width leads to a value of α of

$$\alpha = \frac{jV}{4j_{\text{th}}\theta}\,\frac{(R_{\min}+\Delta)^2}{(R_{\min}+\Delta)^2 + (R_{\min}-R)^2} \qquad (10)$$

where $R_{\min}$ is the distance from the orifice to the keeper plane and Δ is the width of the deposition region.

A one-dimensional spherical code was written that solves for the temperature and potential as a function of radius in the vicinity of the cathode. The plasma density very close to the cathode is calculated from the above fluid equations, Eq. (1a-1c), assuming hemispherical expansion. Numerical solutions are obtained from one-dimensional finite element algorithms. The parameters used in this calculation are from an experiment in Ref. 16. The ion current is 60 mA, and the emitted electron current is 560 mA. The hollow cathode used in the experiment is a ring-cusp plasma contactor.* This

*The ring-cusp plasma contactor is described in Patterson, M. J. and Wilbur, P. J., "Plasma Contactors for Electrodynamic Tether," NASA TM-88850, NASA/AIAA/PSN International Conference on Tethers in Space, Sept. 1986. Similar experiments are described in Ref. 9, and Wilbur, P.J. and Williams, J.D., "An Experimental Investigation of the Plasma Contacting Process," AIAA 25th Aerospace Sciences Meeting, Jan. 1987.

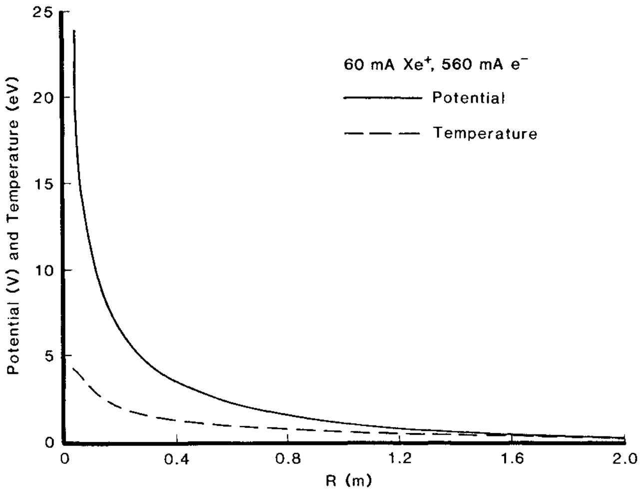

Fig. 2 Temperature and potential profiles of electrons near a hollow cathode electron emitter.

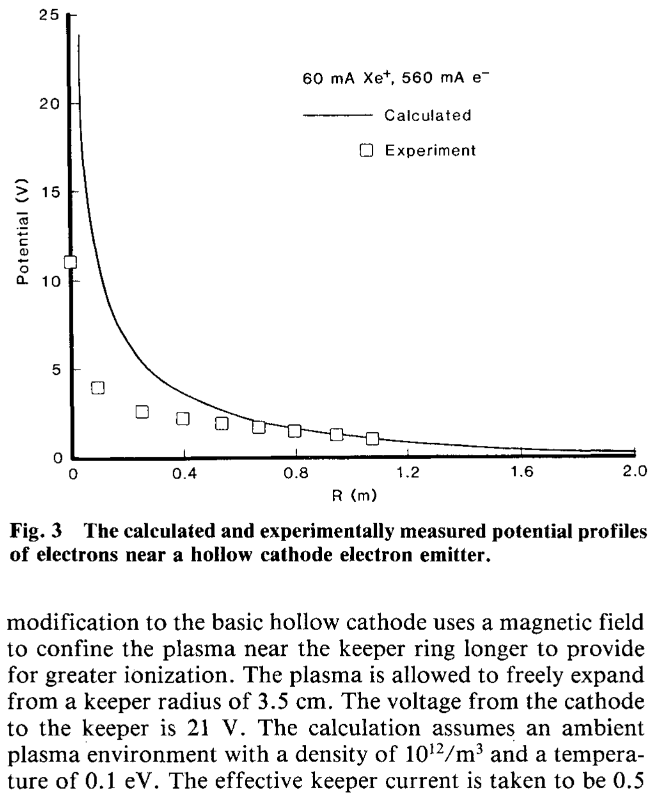

Fig. 3 The calculated and experimentally measured potential profiles of electrons near a hollow cathode electron emitter.

modification to the basic hollow cathode uses a magnetic field to confine the plasma near the keeper ring longer to provide for greater ionization. The plasma is allowed to freely expand from a keeper radius of 3.5 cm. The voltage from the cathode to the keeper is 21 V. The calculation assumes an ambient plasma environment with a density of $10^{12}/m^3$ and a temperature of 0.1 eV. The effective keeper current is taken to be 0.5 amperes, and the deposition thickness is 0.5 cm. Typical temperature and potential profiles that result are shown in Fig. 2. The figure shows that the temperature and potential disturbances fall off very rapidly with distance. Less than a meter away from the cathode, the plasma parameters have fallen to nearly ambient.

Figure 3 shows the comparison between calculation and experiment.[16] The experimentally measured potentials are indicated by the small squares. The magnitude of the experimental potential is about one-half that of the calculations, 11 V compared with 24 V, and of the order of the local electron temperature. This is orders of magnitude smaller than the potentials necessary to emit the same current with an electron gun. The overall shape of the plasma potential curves in Fig. 3 are similar, and the direction of the electric field, *opposing the electron flow*, is the same for both theory and experiment. The direction and magnitude of the electric field are dominated by the need to keep electrons near the cathode to neutralize the charge of the slowly moving ions and are only slightly modified by the fields necessary to support the emitted current. The plasma sheath around the cathode body has not been incorporated into the calculations. Energetic electrons will escape across such a sheath, decreasing the temperature and thus the potentials in the plasma near the orifice.

Calculations were done with different values for each of the parameters. The parameters were chosen so that all but one were the same as in the experiment in order to isolate the effects of each parameter. Most of the parameters do not significantly affect the resulting potential profile as long as reasonable values are used. Variations in the amount of keeper current give only slightly different maximum potentials, e.g., zero keeper current still gives a potential of about 18 V. The most important parameter is the keeper potential. The maximum potential computed is always a few volts greater than the keeper potential. The surprise is that the amount of current emitted is only very slightly dependent upon the voltage. Net currents of 0 to 1.4 amperes have associated voltages of 20–30 V. Above 1.4 amperes the ratio of the electron current to ion current is high enough that the assumptions made in the derivation of Eq. (10) break down.

The major features of these solutions to Eqs. (1a–1c) can be understood by examining the nature of solutions to Eq. (1b). The potential ϕ can be written as a sum of the two terms ϕ_b and ϕ_j where ϕ_b is the solution to

$$\nabla(n\theta) - n\nabla\phi_b = 0 \tag{11a}$$

and ϕ_j is the solution to

$$-n\nabla\phi_j = \eta n j \tag{11b}$$

In *all* of our calculations, using various models for the resistivity, the dominant contribution to the potential is always ϕ_b or the "barometric potential." For a constant temperature, Eq. (11a) can be solved to give the barometric law

$$n = n_o\, e^{-\phi/\theta} \tag{12}$$

where n_o is the ambient plasma density. The barometric behavior of this solution dominates the solution of the full equations even with a nonconstant temperature and a current. For plasma densities above ambient, the potential will always be positive and greater than the temperature. This is because the electrons and ions tend to move toward the region of lowest density, and the electrons can move much faster. The slight positive potential retards the electrons. The high plasma densities in the vicinity of a hollow cathode provide for a strong barometric effect. The net flow of electrons is away from the cathode, toward the region of lower density and against the electric field force. The dominance of the barometric effect means that the most significant aspect of the solution is the temperature; those parameters that affect the temperature at the keeper ring are most important.

Three-Dimensional Calculation of a Hollow Cathode Mounted on the Shuttle

The one-dimensional model describes well the behavior of plasma in the vicinity of the cathode. We now address the expansion of neutralizer ions about a three-dimensional spacecraft. A complete solution to Eqs. (1a–1c) is unnecessary at distances of the order of a meter, since the temperature is near ambient at one meter for even a copious emitter. The barometric law and information on the local density allow us to compute the potential.

A fluid model (HYDRO) of hollow cathode plasmas has been incorporated into NASCAP/LEO,[12] a computer program that simulates in three dimensions the interaction of large, high-power spacecraft with the plasma environment found in low earth orbit. NASCAP/LEO solves Poisson's equation on a three-dimensional cubic grid. Objects can be

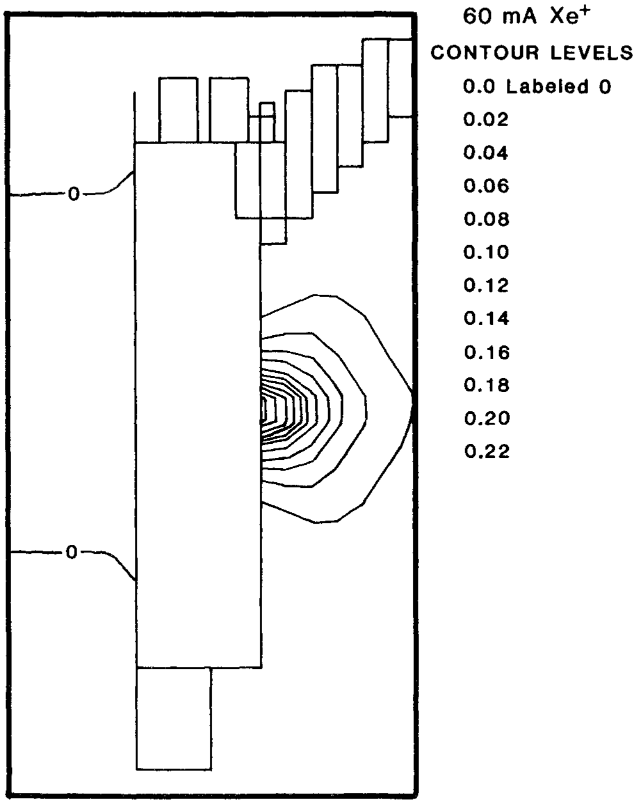

Fig. 4 Potential contours in the vicinity of the shuttle orbiter with a hollow cathode.

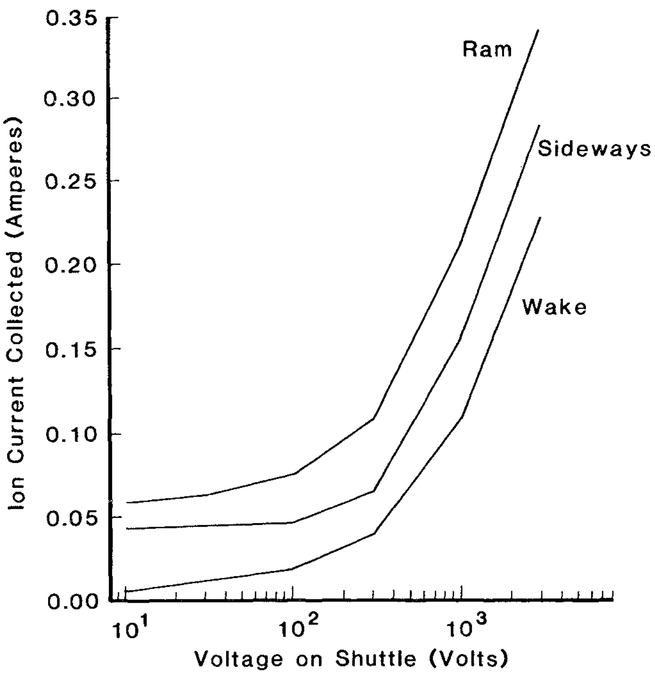

Fig. 5 Ion current collected by the shuttle for three different shuttle orientations.

defined that fit into the cubical grid structure. NASCAP/LEO allows the surfaces of the objects to have various properties that act as boundary conditions to the solution of Poisson's equation. NASCAP/LEO also allows for particle tracking of a limited number of particles.

HYDRO, a module of NASCAP/LEO, uses the LEO grid structure and objects and solves fluid equations for particle motion. HYDRO is designed to solve for the plasma densities and resulting potentials near a hollow cathode, but it can be used for any point source of particles. For a hollow cathode the calculation is divided into two regions. The plasma near the cathode is described with a code that implements the one-dimensional theory described in the previous section. The plasma at distances of a meter or more away from the source is described with the three-dimensional code.

The three-dimensional code solves for the density in all the elements of the three-dimensional grid. The mass continuity equation for ions, Eq. (1a), is solved. The potential is then computed from the barometric law, Eq. (12). The equations are solved from the source outward. The central idea is to adjust the density of particles in each element so that the flux of particles out is equal to the flux into the element. The element with the source in it has a fixed-flux input corresponding to the flux from the cathode. The rest of the elements acquire density from their upwind neighbors. The fluxes are recomputed for the new densities until a convergent solution is reached.

At each iteration, the direction of the particle flow is adjusted. The momentum is shared using the same scheme as is used for mass continuity. An additional term is added in order to include the multiplicity of velocities within each element. The momentum in each element is then adjusted to reflect the acceleration of the particles by their electric fields.

Once self-consistent densities are found, the potentials are calculated according to the barometric law. The ions are absorbed by spacecraft surfaces. Figure 4 shows the potential contours for an electron-emitting hollow cathode mounted on the back of the shuttle. The parameters used in this calculation are the same as those used to generate Figs. 2 and 3. The figure shows the potentials along a slice of the grid through the emitter. Note that the potentials drop quickly and smoothly away from the source and are only a few tenths of a volt. The effects of the return currents to the spacecraft surfaces are not included in this calculation. Therefore, we cannot properly describe differential charging of the spacecraft. Experiments[13] indicate that plasma sources reduce differential charging.

Ion Collection by the Shuttle

For comparison, calculations of the collection of ion currents by the shuttle orbiter were performed using the NASCAP/LEO code. NASCAP/LEO calculations include thermal ion distribution and velocity effects. For several ground potentials, a Poisson calculation of the electrostatic potential was followed by a particle trajectory calculation. The ion current collected consists of those ions that land on the metallic parts of the engine housings. The metallic area of the engine housings was 22.4 m^2 for each of the three main engines and 0.4 m^2 for each of the two OMS engines, totalling 68 m^2 of conductive area. The remainder of the shuttle surface area was assumed to be insulating and at a potential that was slightly negative with respect to plasma ground.

Calculations were run with the engines facing the ram direction (ram position), with the wingspan parallel to the ram direction (sideways position), and with the engines in the wake of the shuttle body and wingspan (wake position). The ion species was taken to be atomic oxygen, and plasma densities of 10^5 cm^{-3} and 10^6 cm^{-3} were used. The plasma temperature was taken as 0.1 eV. The engines in the higher-density plasma collected ten times the current that was collected in the lower density plasma. Results for six logarithmically spaced voltages are shown in Fig. 5. A few example calculations indicated that magnetic field effects are insignificant; if they have any effect, it is to decrease the collection. The collected currents are also reasonably insensitive to the amount of conductive area of the engines. For the engines in the ram direction, we performed calculations on alternative shuttle models having more (96 m^2) and less (10.4 m^2) conductive area. At a potential of -3000 V and density 10^6 cm^{-3}, the low conductive-area model collected 0.19 A, while the 60 m^2 model collected 0.34 A, and the 96 m^2 model collected 0.41 A. Note that even at shuttle voltages of 3000 V the ion current only provides less than one-half of an ampere.

Conclusion

Much progress has been made in understanding hollow cathode-generated plasmas. This understanding allows us to calculate the electron-emitting characteristics of hollow cathodes. The calculated behavior is the same as that observed in the laboratory. Additionally, the calculations indicate that hollow cathodes will behave similarly in space. Using the models described in this paper, the calculations indicate that a hollow cathode can be used to maintain the shuttle potential near plasma ground for tether currents up to one ampere. This is in contrast to electron guns, such as those planned for TSS-1, that require thousand-volt accelerating potentials for ampere currents and collection of ambient ions, which NASCAP/LEO calculations show would require higher potentials than could be generated by the tether.

Acknowledgment

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